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Directed transport in quantum Hall bilayers

R. Khomeriki[1,a], L. Tkeshelashvili[2,3], T. Buishvili[1], and Sh. Revishvili[1]

[1] Department of Natural Sciences, Tbilisi State University, Chavchavadze ave. 3, Tbilisi 0128, Georgia
[2] Institut für Theoretische Festkörperphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany
[3] Institute of Physics, Tamarashvili str. 6, Tbilisi 0177, Georgia

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Abstract. The pseudo-spin model for a double layer quantum Hall system with the total landau level filling factor $\nu = 1$ is discussed. In contrast to the "traditional" model where the interlayer voltage enters as a static magnetic field along pseudo-spin hard axis, taking into account the realistic experimental situation, in our model we interpret the influence of applied voltage as a source of additional relaxation process in the double layer system. We show that the Landau-Lifshitz equation for the considered pseudo-magnetic system well describes existing experimental data and reduces to the dc driven and damped sine-Gordon equation. As a result, the mentioned model predicts novel directed intra-layer transport phenomenon in the system. In particular, unidirectional intra-layer energy transport can be realized due to the motion of topological kinks induced by applied voltage. Experimentally this should be manifested as counter-propagating intra-layer inhomogeneous charge currents proportional to the interlayer voltage and total topological charge of the pseudo-spin system.

PACS. 73.43.Lp Collective excitations – 05.45.Yv Solitons

1 Introduction

The anomalous transport and tunneling properties of quantum Hall bilayers with the total Landau level filling factor $\nu = 1$ results in a steady interest in such systems during the recent years [1–11]. Quantum Hall bilayers consist of electrons confined in closely separated two dimensional semiconductor layers in applied high magnetic fields. In the absence of interlayer voltage, each layer of the system has a filling factor $\nu_1 = \nu_2 = 1/2$. Since the layers are identical, the system can be phenomenologically described via the pseudo-spin formalism [2]. In particular, an electron in one layer acquires the pseudo-spin pointing "up", while in the other layer it has the pseudo-spin pointing "down". The z component of the overall pseudo-spin vector specifies the charge imbalance between the layers. It is clear that the system has a lower energy when the pseudo-spin points neither up nor down, but rather lies in the plane, reflecting the fact that in the ground state electrons are equally distributed between the two layers. Therefore, within this formalism, double layer quantum Hall system is treated as an easy plane ferromagnet with a hard axis anisotropy and an electron tunneling between the layers corresponds to the spin flips in the pseudo-spin language.

This is a quite satisfactory model for isolated double layer systems. However, problems start to arise as soon as one considers a real experimental situation with applied interlayer voltage and induced tunneling current. Traditionally, in the phenomenological Hamiltonian, interlayer dc voltage is interpreted as a constant magnetic field applied along z axis [2]. Although this model correctly describes the physics at low interlayer voltages [3], it fails to describe experimentally observed current-voltage characteristics [1] for large interlayer voltages. The point is that, the introduction of interlayer voltage as a homogeneous static magnetic field is correct if the voltage is uniformly applied along a whole bilayer. However, in the real experimental situation [1] the contacts are connected at the edge of the double layer. Therefore, we propose here to consider the two subsystems. One of the subsystems consists of the bilayer with zero interlayer voltage which is connected with the second subsystem, represented by the bilayer "reservoir", with parameters defined by the external leads. In addition we note that while the charge imbalance of the bilayer right at the contact is solely defined by the applied interlayer voltage, the charge imbalance of the rest of the bilayer simply is forced to relax to this nonequilibrium state. It is clear that the relaxation rate is proportional to the sheet-contact conductivity.

As we shall demonstrate below, the proposed relaxation mechanism well describes experimental observations by Spielman et al. [1]. In particular, the equations of motion in certain limit reduce to a generalized sine-Gordon equation which, in the context of quantum Hall bilayers,

[a] e-mail: khomeriki@hotmail.com

was introduced in references [4,12,13]. On the other side the sine-Gordon model leads to the prediction of directed solitonic transport phenomenon [14]. This exotic effect can be directly verified in the experiment.

In this paper we present the symmetry analysis [14] and numerical simulations in order to show that the directed inhomogeneous intra-layer current appears due to propagation of topological excitations in a quantum Hall bilayer. Moreover, we suggest the realistic experimental set-up in order to observe this phenomenon.

2 The effective model

The effective Hamiltonian density of our phenomenological model of double layer quantum Hall (pseudo) ferromagnet is given by:

$$\mathcal{H} = \frac{\rho_E}{2}\left(\frac{\partial m_x}{\partial x}\right)^2 + \frac{\rho_E}{2}\left(\frac{\partial m_y}{\partial x}\right)^2 + \beta m_z^2 - \frac{\Delta_{SAS}}{2} m_x \quad (1)$$

where the unit vector $\boldsymbol{m}(x,t)$ is an order parameter; ($m_z = \nu_1 - \nu_2$ describes local electrical charge imbalance between the two layers); ρ_E is the in-plane spin stiffness, β gives a hard axis anisotropy and Δ_{SAS} is a tunneling amplitude. Then Landau-Lifshitz equations of motion, which conserve the length of the local spin density, can be derived as follows:

$$\frac{\partial \boldsymbol{m}}{\partial t} = [\boldsymbol{m} \times \boldsymbol{H}], \quad \boldsymbol{H} = -2\left\{\frac{\partial \mathcal{H}}{\partial \boldsymbol{m}} - \frac{\partial}{\partial x}\left[\frac{\partial \mathcal{H}}{\partial \frac{\partial \boldsymbol{m}}{\partial x}}\right]\right\} \quad (2)$$

where $\boldsymbol{H}$ is the effective pseudo-magnetic field.

We will consider below only the case of small charge imbalances between the layers, which allows for further reduction of the equations of motion. Particularly, in the limit $m_z \to 0$, one can rewrite equation (2) as follows:

$$\frac{\partial m_z}{\partial t} = 2\rho_E \frac{\partial^2 \varphi}{\partial x^2} - 2\Delta_{SAS}\sin(\varphi) \quad (3)$$

$$\frac{\partial \varphi}{\partial t} = 4\beta m_z, \quad (4)$$

where the phase variable φ is defined from the relation $m_x + i m_y = \sqrt{1 - m_z^2}\exp(i\varphi)$. Reminding that m_z describes a local charge imbalance between the layers, equation (3) could be interpreted as the charge continuity equation with a damping term, where

$$J_S = -2e\rho_E \frac{\partial \varphi}{\partial x}, \quad J_{tun} = 2e\Delta_{SAS}\sin(\varphi) \quad (5)$$

are intra-layer current in each layer and interlayer tunneling current, respectively.

Now we must take into account the relaxation process announced at the beginning. As it was already noted, the bilayer near the contacts is described by the same Hamiltonian (1) adding just the Zeeman term $\omega m_z/2$, where $\omega = eV/\hbar$ includes interlayer voltage V and e is an electron charge. Then, in the stationary state the charge imbalance of that part of the bilayer is expressed as

$$m_z^0 = \omega/4\beta. \quad (6)$$

The rest part of the bilayer (i.e. far from the contacts) will relax to this nonequilibrium value with the relaxation rate $1/R$, where R is a sheet+contact resistance. Therefore, we have to modify equations (3) and (4) by adding the relaxation term as follows:

$$\frac{4\beta}{R}\left(m_z - m_z^0\right) + \frac{\partial m_z}{\partial t} = 2\rho_E \frac{\partial^2 \varphi}{\partial x^2} - 2\Delta_{SAS}\sin(\varphi) \quad (7)$$

$$\frac{\partial \varphi}{\partial t} = 4\beta m_z. \quad (8)$$

Further, substituting (8) into the continuity equation (7), we finally obtain:

$$\frac{1}{R}\frac{\partial \varphi}{\partial t} - \frac{\omega}{R} + \frac{1}{4\beta}\frac{\partial^2 \varphi}{\partial t^2} - 2\rho_E \frac{\partial^2 \varphi}{\partial x^2} + 2\Delta_{SAS}\sin(\varphi) = 0. \quad (9)$$

Without the gradient term this equation for the first time was suggested by Wen and Zee [12] (see also Refs. [4,13]). The results of reference [12], as well as our numerical simulations, suggest that there are completely different regimes of the pseudo-spin dynamics for the high and low voltages. In particular, in the first case $V \to 0$ the analytical solution of (9) should be sought as:

$$\varphi = \phi_0, \quad (10)$$

where ϕ_0 is a constant quantity. Substituting (10) into the equation of motion (9) we can define ϕ_0 as follows:

$$2\Delta_{SAS}\sin\phi_0 = \omega/R. \quad (11)$$

Moreover, from (5) the expression for the tunneling current reads:

$$J_{tun} = (e^2/\hbar)(V/R). \quad (12)$$

The solution (11) defines the limits for the high and low voltage regimes. Particularly, the solution (10), (11) holds in the low voltage limit ($eV/\hbar \ll 2\Delta_{SAS}R$), while in case of high voltages ($eV/\hbar \gg 2\Delta_{SAS}R$) the solution (10), (11) cannot be used any more (since, in this limit, $\sin\phi_0$ in (11) becomes larger than one).

For the case of high voltages the solution is sought as the following combination:

$$\varphi = \omega t + A\sin(\phi_0 + \omega t), \qquad A \ll 1, \quad (13)$$

where the constants A and ϕ_0 must be defined perturbatively by substituting (13) into the equation of motion (9) (see Refs. [4,16]). As a result we obtain:

$$\tan\phi_0 = \frac{4\beta}{\omega R}, \qquad A = \frac{8\beta/R}{\omega\sqrt{\omega^2 + (4\beta/R)^2}}, \quad (14)$$

and the dc component of tunneling current is given by:

$$J_{tun} = 2e\Delta_{SAS}\overline{\sin\left[A\sin\left(\phi_0 + \omega t\right) + \omega t\right]} =$$
$$e\Delta_{SAS}A\sin\phi_0 = \frac{e^2}{\hbar}V\frac{32\Delta_{SAS}^2\beta^2/R}{(eV/\hbar)^2\left[(eV/\hbar)^2 + (4\beta/R)^2\right]}. \quad (15)$$

It is easy to see that equations (12) and (15) qualitatively well describe the experimentally observed tunneling current-voltage characteristics [1]. Indeed, at the low voltages the current increases with voltage, while for the high voltages the tunneling current decreases as $1/V^3$.

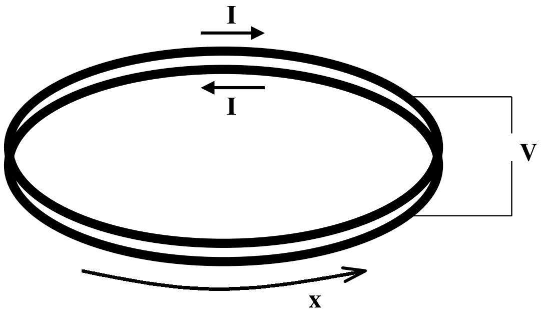

Fig. 1. Suggested experimental setup for observation of inhomogeneous counter propagating intra-layer charge currents by applying an interlayer dc voltage on a quantum Hall bilayer.

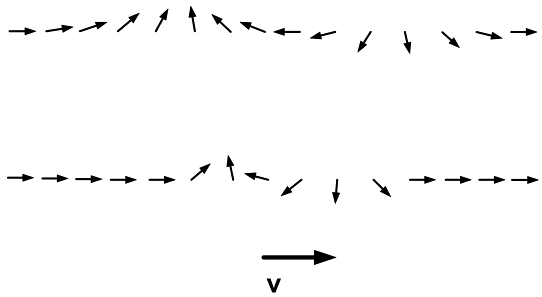

Fig. 2. Upper panel: schematic presentation of initial pseudospin distribution with topological charge $Q = 1$. Lower panel shows the pseudospin system behavior after application of interlayer voltage. The topological charge becomes localized and starts to move along the layer with a velocity proportional to the applied interlayer voltage.

3 Symmetry analysis and numerical estimates

In the consideration given above it was assumed that the total topological charge of the system is zero. Consequently, we obtain that the intra-layer current is zero for any voltages according to the definitions (5). The situation, however, drastically changes if nonzero topological charge is present in the system.

In order to carry out the symmetry analysis, we should write down the expression for the density of energy flux in the system (see e.g. Ref. [14]):

$$J_E = -\rho_E \frac{\partial \varphi}{\partial x}\frac{\partial \varphi}{\partial t} \tag{16}$$

and define topological charge of the system as follows:

$$Q = \Big[\varphi(+\infty) - \varphi(-\infty)\Big]/2\pi. \tag{17}$$

In general, the topological charge Q may take any integer value. For simplicity, in our numerical experiment we choose an initial topological charge as $Q = 1$. Therefore, we take (see Fig. 2):

$$\varphi(x) = 2\pi x/L, \qquad m_z(x) = 0, \tag{18}$$

where L is a length of double layer "ring" (see Fig. 1). According to the general approach [14] let us consider the following symmetry transformations:

$$x \to -x, \qquad \varphi \to -\varphi. \tag{19}$$

These symmetry transformations leave the topological charge (17) invariant but change the sign of the density of energy flux (16). In this case the symmetry properties of equations of motion becomes crucial. If the equations are invariant under the symmetry transformations (19) the averaged energy flux in the system is identically zero, otherwise directed energy flux will exist [14]. In the present case the equations of motion (9) are not invariant with respect the symmetry transform (19). As a consequence, this leads to the directed intra-layer energy transport for any applied nonzero interlayer voltage. Similarly, for zero voltage equation (9) is invariant under the symmetry transformation (19) and thus averaged energy flux should be zero.

The numerical analysis of the motion equation (9) completely agrees with the predictions of the symmetry analysis. Thus, in the presence of nonzero total topological charge in the system, directed energy transport should be observed. Moreover, the direction of the transport can be changed by inverting the sign of the voltage. Moreover, we can further extend the analytical consideration noting that the equation of motion (9) is nothing but dc driven-damped sine-Gordon equation (see e.g. Refs. [17,18]). The role of the dc driven force plays the term $f = 4\beta\omega/R$. As is well known, sine-Gordon equation in the absence of driving force supports solitary wave solutions with nonzero topological charge. These solutions often are termed as kinks and are given by the following expression:

$$\varphi = 4\arctan\left\{\exp\left[\sqrt{\frac{\Delta_{SAS}}{\rho_E}}(x - x_0)\right]\right\}. \tag{20}$$

Applied driving force leads to the motion of such excitations, and the velocity of motion is proportional to the driving force (applied voltage). Thus, by increasing the voltage, the intra-layer energy transport (and consequently the inhomogeneous charge current) can be increased.

We have performed the numerical simulations with the set of realistic physical parameters usually used in the experiments [1,5,6]: $\beta = 7$ K, $\rho_E = 1.6 \times 10^{-16}$ K m^2, $\Delta_{SAS} = 10^{-4}$ K, $1/R = 10^{-3}$ K and we choose the value of interlayer voltage $V = 2 \times 10^{-5}$ V. The initial topological charge of the system equals to one and the periodical boundary conditions are used in all numerical experiments. The results are presented in Figures 3, 4 as space-time evolution of local charge imbalance and local charge density. As seen, topological kink forms and moves with a constant velocity creating the inhomogeneous counter propagating charge currents in each layer.

4 Conclusion

In the present paper the driven-damped pseudo-magnetic model is elaborated in order to describe the dynamics in

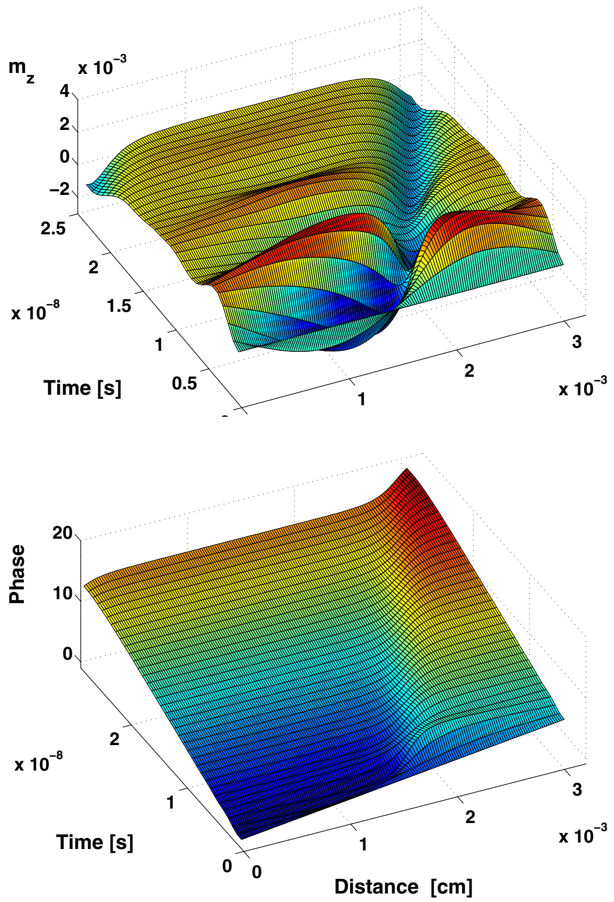

Fig. 3. Evolution of the local charge imbalance m_z (upper graph) and phase (lower graph) along the quantum Hall bars. Besides the creation of charge imbalance, interlayer voltage induces the motion of localized excitations in case of topologically nontrivial initial conditions.

quantum Hall bilayers at the total filling factor $\nu = 1$. By means of the symmetry analysis and numerical simulations it is shown that the directed transport exists in the system in the presence of nonzero topological charge. Moreover, the realistic experimental setup is suggested for observing the suggested phenomenon. Initial nontrivial topological charge [like presented in Exp. (18)] in the system could be realized in the laboratory experiments by application of a weak in-plane magnetic field along the double layer "ring" (x direction in Fig. 1), for which a commensurate pseudo-spin distribution appears. Then, by switching off the in-plane field and applying an interlayer dc voltage it will be possible to observe the inhomogeneous counter propagating currents in each layer (see Fig. 2). However, we note that the thermal fluctuations will reduce the topological charge in a double layer system (like in case of narrow superconducting channels [19]) and as a result the intra-layer current eventually should decrease as well.

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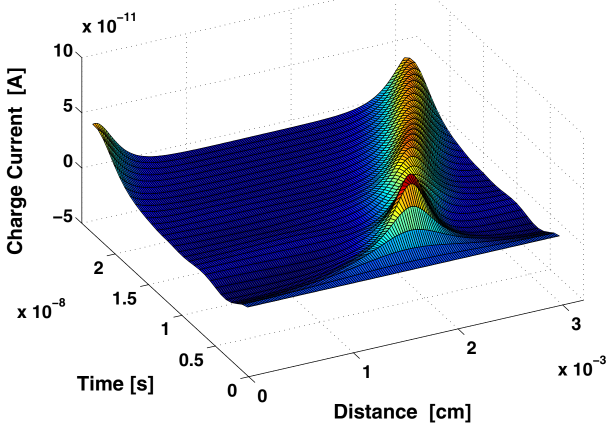

Fig. 4. Intra-layer local charge current versus time and distance in the case of nonzero initial topological charge ($Q = 1$) and nonzero interlayer voltage. In the numerical experiments the periodic boundary conditions are used.

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